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Scaling laws in the spatial structure of urban road networks

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Abstract

The urban road networks of the 20 largest German cities have been analysed, based on a detailed database providing the geographical positions as well as the travel-times for network sizes up to 37,000 nodes and 87,000 links. As the human driver recognises travel-times rather than distances, faster roads appear to be 'shorter' than slower ones. The resulting metric space has an effective dimension $\delta > 2$, which is a significant measure of the heterogeneity of road speeds. We found that traffic strongly concentrates on only a small fraction of the roads. The distribution of vehicular flows over the roads obeys a power law, indicating a clear hierarchical order of the roads. Studying the cellular structure of the areas enclosed by the roads, the distribution of cell sizes is scale invariant as well.

Keywords: Urban road network; Graph topology; Power-law scaling; Travel-times; Vehicle traffic; Cellular structure; Effective dimension; Hierarchy

1. Introduction

The scientific interest in network analysis has been steadily growing since the revolutionary discoveries of Watts and Strogatz [1] and Barabási and Albert [2]. They found out that many real-world networks such as the internet and social networks exhibit a scale-free structure characterised by a high clustering coefficient and small average path lengths. The path lengths, however, are usually not related to geographical distances. Surprisingly, little attention has been paid to the spatial structure of networks, even though distances are very crucial for logistic, geographical and transportation networks.

Urban road networks with links and nodes representing road segments and junctions, respectively, exhibit unique features different from other classes of networks [3–8]. As they are almost planar, they show a very limited range of node degrees. Thus, they can never be scale-free like airline networks or the internet [5]. Nevertheless, there exists an interesting connection between these scale-free networks on the one hand and road networks on the other hand, since both are extreme cases of an optimisation process minimising average travel costs along all shortest paths, given a set of nodes and a total link length. The properties of the resulting networks strongly depend on the links' cost function. If the travel costs on all links were equal, small-world networks with a hub-and-spoke architecture typical for airline networks or the internet would emerge.

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However, with travel costs proportional to the link length, the resulting networks would exhibit properties typical for road networks [5].

We have extracted road network data of the administrative areas of the 20 largest German cities from the geographical database Tele Atlas MultiNet^{TM} [9], typically used for real-time navigation systems or urban planning and management. The data provide a geo-coded polygon for each road segment as well as a series of properties, e.g. the length, the average expected travel-time, the speed limit, the driving directions, etc. Junctions and homogeneous road segments of the selected areas are represented by nodes and links of a corresponding directed graph. The location of the cities within Germany and the corresponding networks sizes are shown in Fig. 1 and Table 1. Since the road network of Hanover, ranked 11th, could not be extracted unambiguously, it was excluded from our analysis.

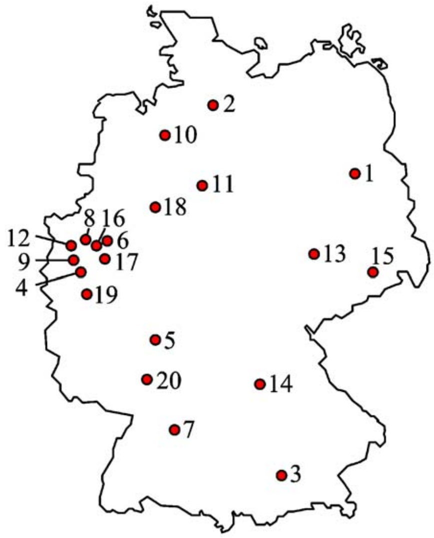

Fig. 1. Germany with it's 20 largest cities, ranked by their population.

Table 1
The 20 largest cities of Germany and their characteristic coefficients referred to in the following sections

No.	City	Population in 1000	Area (km^2)	No. of nodes	No. of links	Fraction of trees (%)	Effective dimension (δ)	Betweenness exponent (β)	Gini index (g)	Cell size exponent (α)	Form factor variance (s_ϕ)
1	Berlin	3392	891	37,020	87,795	11.55	2.330	1.481	0.871	2.158	0.159
2	Hamburg	1729	753	19,717	43,819	11.93	2.350	1.469	0.869	1.890	0.164
3	Munich	1235	311	21,393	49,521	10.74	2.463	1.486	0.869	2.114	0.159
4	Cologne	969	405	14,553	29,359	21.27	2.372	1.384	0.875	1.922	0.165
5	Frankfurt	644	249	9728	18,104	16.90	2.388	1.406	0.873	2.009	0.169
6	Dortmund	591	281	10,326	22,579	22.82	2.091	1.340	0.875	1.809	0.166
7	Stuttgart	588	208	10,302	21,934	23.30	2.008	1.377	0.894	1.901	0.170
8	Essen	585	210	11,387	24,537	22.80	2.243	1.368	0.892	1.932	0.169
9	Düsseldorf	572	218	8237	16,773	19.35	2.700	1.380	0.849	1.964	0.175
10	Bremen	543	318	10,227	21,702	23.98	2.220	1.351	0.909	1.931	0.166
11	Hanover	517	204	1589	3463	–	–	–	–	–	–
12	Duisburg	509	233	6300	14,333	17.57	2.050	1.480	0.900	1.924	0.169
13	Leipzig	495	293	9071	21,199	6.78	2.304	1.320	0.880	1.926	0.153
14	Nuremberg	493	187	8768	18,639	19.68	2.399	1.420	0.854	1.831	0.172
15	Dresden	480	328	9643	22,307	20.45	2.205	1.355	0.870	1.892	0.156
16	Bochum	389	146	6970	15,091	22.19	2.279	1.337	0.847	1.829	0.171
17	Wuppertal	364	168	5681	11,847	27.75	2.040	1.279	0.881	1.883	0.162
18	Bielefeld	325	259	8259	18,280	26.44	2.337	1.337	0.872	1.735	0.161
19	Bonn	309	141	6365	13,746	25.73	2.134	1.374	0.889	2.018	0.173
20	Mannheim	309	145	5819	12,581	17.79	2.114	1.455	0.897	1.959	0.162

2. Effective dimension

In transportation networks with strong geographical constraints, it is observed that the sizes of neighbourhoods grow according to a power law [10]. We study properties and implications of such scaling in urban road networks, where distances are, with respect to human driver's recognition, related to travel times. Human travel behaviour underlies the universal law of a constant energy budget [11]. The cost of travel must, therefore, not be measured in the number of road meters, but in the amount of energy or, assuming a single mode of transport with a constant energy consumption rate, e.g. car driving, in units of travel-time. Interestingly, this implies that routes along faster roads appear 'shorter' than slower ones in terms of travel-time. A distant but well accessible destination is virtually closer than a near one with a longer time to access. The virtual compression of faster and the dilation of slower roads result in an effective deformation of the urban space, whose metric structure we are going to study.

For any node in the road network, the number of nodes reachable with a given travel-*distance* budget r, i.e., the number of nodes to which the shortest path is shorter than r, essentially scales with a maximum exponent of 2. This fact is independent of whether the graph is planar in a strict sense or the urban landscape is uneven. Considering shortest paths with respect to travel-*time* instead, the number of nodes $N_v(\tau)$ reachable with a travel-*time* budget τ follows a scaling law $N_v(\tau) \sim \tau^{\delta}$ with δ being significantly larger than 2 for all road networks under consideration, see Table 1. The scaling exponent δ is called the effective dimension [5]. The existence of arterial roads with road speeds above average allows car drivers to reach distant places over-proportionally fast, which results in higher values of δ. Thus, the effective dimension can be used as a measure of the heterogeneity of the road speeds. Fig. 1(a) shows the areas reachable from a central node within different travel-time budgets.

Referring to Refs. [5,10], the effective dimension δ is theoretically defined by

$$\delta = \lim_{\tau \to \infty} \frac{d \log N_v(\tau)}{d \log \tau}. \tag{1}$$

Since road networks are finite, however, this formula has to be approximated. For all nodes we have computed the average $\overline{N}_v(\tau)$ and plotted it over τ in double logarithmic scale as shown in Fig. 2(b). For larger values of τ, the curve saturates due to the finite number of nodes in the graph. The slope of this curve at its inflection point gives the lower bound for an estimation of δ (dotted line). Alternatively, one could also periodically continue the graph, e.g. by mirroring a rectangular part of it and estimate the limit for $\tau \to \infty$.

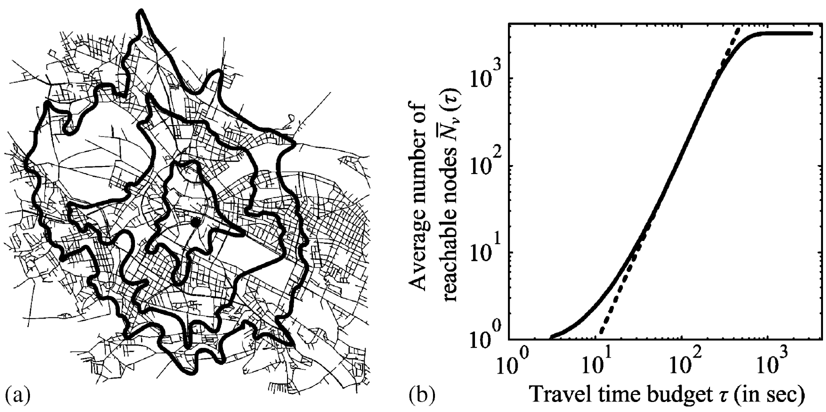

Fig. 2. (a) Isochrones (bold lines) surround areas reachable from a point in the city centre of Dresden with different travel-time budgets $\tau_1{:}\,\tau_2{:}\,\tau_3 = 1{:}\,2{:}\,3$. These areas extend wider along fast roads, e.g. in the north, while they are compressed along slower roads, e.g. in the east. (b) Average number $\overline{N}_v(\tau)$ of nodes reachable within a travel-time budget τ.

3. Distribution of traffic

The heterogeneity of road speeds also has an impact on the distribution of vehicular traffic in the road network. Faster roads are more attractive for human drivers, resulting in a concentration of traffic along these roads, see Fig. 3.

The importance of a road or a junction can be characterised by the number of cars passing through it within some time interval. This can roughly be approximated with the measure of link betweenness centrality b_e and node betweenness centrality b_v. It is given by the number of shortest paths with respect to travel-time between all pairs of nodes in the corresponding graph, of which the particular link e or node v is part of [7,8,12–14]. Using the measure of betweenness centrality holds, we assume equally distributed origin–destination pairs, identical average departure rates, and the absence of externalities. Even though these assumptions might not hold for precise traffic flow predictions, they allow for estimating the implications of the network topology on the spatial distribution of traffic flows.

The German road networks show an extremely high node betweenness centrality b_v at only a small number of nodes, while its values are very low at the majority of nodes. Fig. 4(a) shows the distribution of its relative frequency density $p(b_v)$. Over the entire range, the distribution follows the scale-free power law $p(b_v) \sim b_v^{-\beta}$ with the exponent $\beta = 1.355$ for Dresden, see also Table 1. High values of β can be interpreted as a high concentration of traffic on the most important intersections.

Studying the link betweenness centrality b_e reveals a similar picture: the traffic volume is highly concentrated on only a few roads, or to be more precise, on only a few road meters. By referring to road meters instead of roads we overcome the effect of different road lengths. As a quantitative concentration measure we use the Gini index g, which can be obtained from the Lorenz curve [15]. The Lorenz curve is an monotonously increasing and convex curve joining the points (F, P), where F is the fraction of all road meters that have a fraction P of the total length of all shortest paths leading over it. The Gini index g is defined as twice the area between the Lorenz curve and the diagonal. In the extreme case of a perfect equal distribution, the Lorenz curve would follow the diagonal with $g = 0$. In the other extreme case of a distribution similar to delta function, we would find $P = 0$ for all $F < 1$, and $P = 1$ if $F = 1$, and the Gini index would be $g = 1$. The Lorenz curve for the road network of Dresden is shown in Fig. 4(b) and can be interpreted as follows: 50% of all road meters carry as little as 0.2% of the total traffic volume only (I), while almost 80% of the total traffic volume are concentrated on no more than 10% of the roads (II). Most interestingly, half of the total traffic volume is handled by only 3.2% of the roads in the network (III). The related Gini index of Dresden is $g = 0.870$, see Table 1.

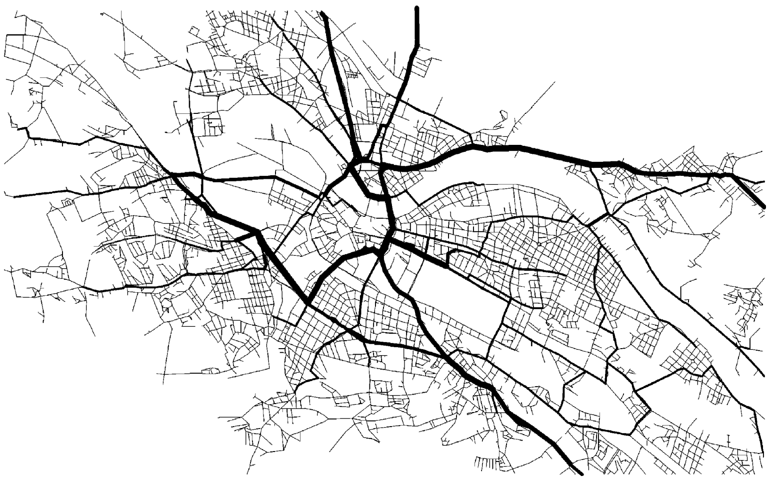

Fig. 3. Shortest paths in the road network of Dresden. The width of the links corresponds to the respective betweenness centrality b_e, that is an approximate measure of the amount of traffic on that roads.

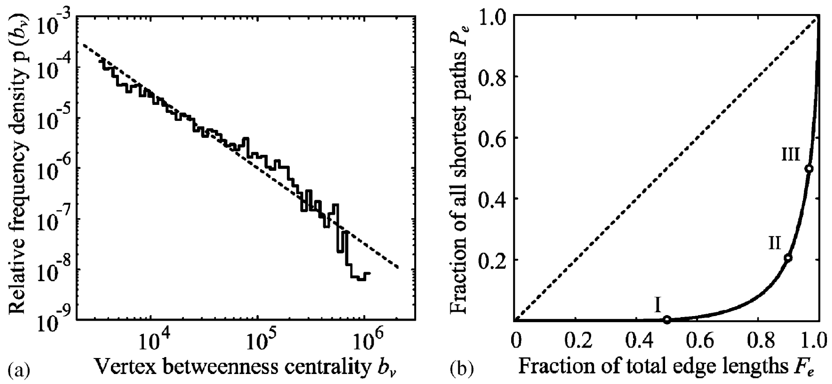

Fig. 4. (a) The distribution of the node betweenness centrality b_v obeys the power law $p(b_v) \sim b_v^{-\beta}$ with the exponent $\beta = 1.355$ for Dresden (dotted line). (b) The Lorenz curve (solid line) for Dresden.

The bundling of traffic streams on a few arterial roads reflects the clear hierarchical structure of the roads. The existence of hierarchies is an inherent property of transportation networks [16]. Fig. 3 shows that the arterial roads sprawl out from the city centre in all direction of the network.

Besides the diversity of road speeds, the inherent structure of the road network topology itself has a tremendous effect on the emergence of road hierarchies. Dead-end roads, for example, are at the lowest level of the road hierarchy by definition. Interestingly, the fraction of dead-end roads or, more precisely, the fraction of tree-like structures in the corresponding graph is about 20% of the total road length in the network of Dresden. Some of the dead-ends may belong to the boundary of the road network, but their fraction should be small since only a few country roads or highways are cut. Such tree-like structures, also referred to as 'lollipop' layouts, are typical for modern North American suburbs [8] and are found among the 20 German cities under consideration as well.

4. Cellular structures

The structure and spatial extension of trail systems [17,18] is constrained by the presence of impenetrable or inaccessible places. The structure of road networks, therefore, is a result of an interplay between travel cost minimisation and efficient land use. Facilities, residences, parks, etc. are enclosed by the roads, letting the road network appear as a two-dimensional cellular system. Such structures are typical for trail systems as well as for self-generated structures like crack patterns, maple leaves, dragonfly wings, etc. [19].

The topology of two-dimensional cellular structures has been studied in the domain of planar graph theory [20,21] since Euler, whose theorem states that the number N_c of bounded cells in a connected planar graph with N_v nodes and N_e undirected links is given with $N_c = N_e - N_v + 1$. The graph of road networks is always connected but, due to the presence of bridges and tunnels, obviously not planar in a strict sense, as is required for the definition of cells. Thus, for our investigations, we determined all pairs of crossing links and connected them by adding virtual nodes at the crossing points.

A cell's neighbourhood degree k_c is the number of adjacent cells [21] or, which is equal to that, the number of non-dead-end roads the cell is connected to. The frequency distribution $P(k_c)$ of neighbourhood degrees for the road network of Dresden is shown in Fig. 5(a). In all 20 road networks under consideration, around 80% of the cells have three to six neighbours, where those with four neighbours are always predominating. This is in perfect agreement with the observations in non-planned settlements, while in crack patterns and maple leaves the most frequent neighbourhood degree is always five, in dragonfly wings and honey combs it is even six [19]. This leads to the conjecture, that the most frequent neighbourhood degree of four is a distinctive feature of urban road networks. The frequency density distribution $p(A_c)$ of the surface areas A_c is shown in Fig. 5(b). Note that we neglected cells of size smaller than $10,000\,\mathrm{m}^2$, which are usually artefacts of the data's high precision, obviously representing vacancies within more complicated intersection layouts. The

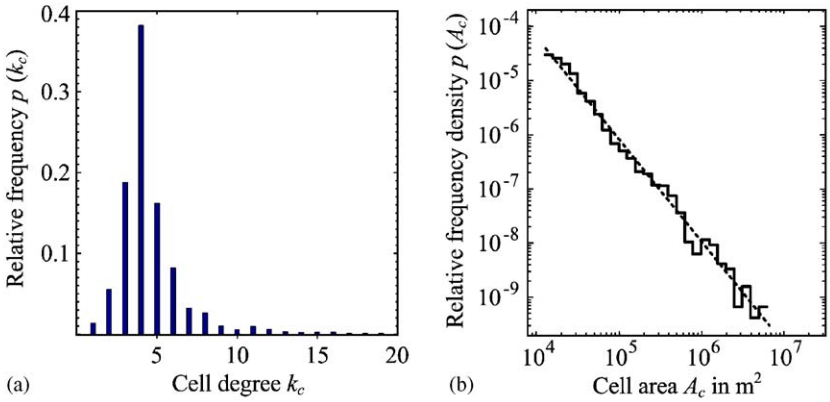

Fig. 5. (a) Frequency distribution of neighbourhood degrees k_c of the cells in the road network of Dresden. The predomination of cells with four neighbours was found in all 20 German road networks. (b) The frequency distribution of the cell's surface areas A_c obeys the scale-free power law $P(A_c) \sim A_c^{-\alpha}$ (dotted line) with the exponent $\alpha = 1.892$ for the road network of Dresden.

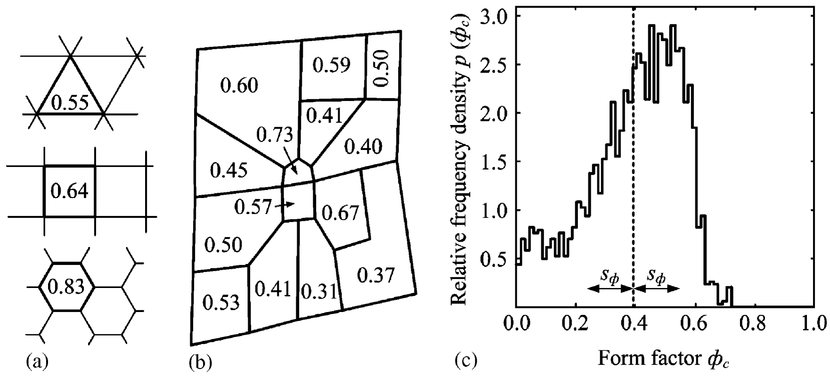

Fig. 6. Form factor ϕ_c of cells in (a) regular structures and (b) in the district of Striesen in the road network of Dresden. (c) The frequency density distribution $p(\phi_c)$ of the form factors has a standard deviation (small arrows) of $s_\phi = 0.156$ indicating a broad diversity of the cell shapes. The dotted line represents the mean value $\bar{\phi}_c$.

distribution $p(A_c)$ is scale invariant and obeys the power law $p(A_c) \sim A_c^{-\alpha}$ with the exponent $\alpha = 1.892$ for the road network of Dresden, see Table 1.

As a quantitative measure of the compactness or roundness of a cell c, we use the form factor ϕ_c. It is the fraction of the surface area of the circumscribed circle that is covered by the cell. With D_c denoting the maximum distance between two points on the boundary of the cell, the form factor can be estimated by $\phi_c = 4/\pi\left(A_c/D_c^2\right)$. The values of ϕ_c range from 0 to 1 and correspond to the limiting cases of infinitely narrow and perfectly circular cells, respectively. Fig. 6(a) gives an example of form factors in homogeneous grid structures and Fig. 6(b) shows a small part of the road network of Dresden. The frequency density distribution $p(\phi_c)$ of form factors in the road network of Dresden is shown in Fig. 5(c). The maximum value found is $\phi_c = 0.73$, while 70% of the cells have a form factor in the range between 0.3 and 0.6. The standard deviation of $s_\phi = 0.156$, see Table 1, reflects a broad diversity of cell shapes. This might result from the long history of German cities, that were growing over several centuries and contain both, historic centres and modern regularly structured areas designed according to today's infrastructural demands.

5. Summary

We have analysed real-world data of urban road networks of the 20 largest German cities. Considering travel-times rather than distances reveals an effective dimension significantly larger than two. Centrality measures allow for the quantification of 'important' or frequently used road segments and reflect the hierarchical structure of the road network. The shape of the cells encircled by road segments can be quantified through the notion of a form factor. We found scaling of several aspects of road networks, such as the distribution of cell sizes or the number of nodes reachable within a travel-time budget. In contrast to many material transport networks in biology such as vascular [22], however, their topological organisation is less obvious and a hierarchical structure similar to a Cayley tree is not found at all.

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